INDIAN INSTITUTE OF SPACE SCIENCE & TECHNOLOGY

B. Tech(I Year) Physics - II (PH121) Quiz 1

15 Feb' 2017

Duration:1 Hrs

Full Marks: 30

Answer all questions (All questions carry equal marks)

- 1. If $\mathbf{A} = \frac{1}{2} (\mathbf{B} \times \mathbf{r})$, where **B** is a constant vector, find $\nabla \times \mathbf{A}$.
- 2. Find the curl of the vector function $\mathbf{B}(s, \theta, z) = \frac{1}{s}\hat{\theta}$ (expressed in cylindrical coordinates). Note: The curl must be consistent with the Stokes' theorem (say, around a circular loop centered at the origin).
- 3. Show that the integral

$$J = \int_{V} \operatorname{sech} \lambda \mathbf{r} \left(\nabla \cdot \frac{\hat{\mathbf{r}}}{\mathbf{r}^{2}} \right) \mathrm{d}\tau$$

evaluated using (a) Gauss theorem and (b) Dirac's delta function gives the same result. Here, V is a sphere of radius R centered at origin, and λ is a constant.

4. Check the Stokes theorem for the function $\mathbf{V} = y\hat{\mathbf{x}}$, for the path shown in the figure below, and the surface composed of the three flat shaded areas. The curved part is a circle of radius 'a', centered at the origin.

- 5. A charge distribution with spherical symmetry has density where k is a constant.
 - a) Determine the electric field everywhere.
 - b) Find the work done in assembling the charge configuration.

c) Find the energy contained in the electric field outside the sphere due to this configuration.

6. Four charges -Q, 2Q, -3Q, and 4Q, are placed, in that order, on the four corners of a square of size 'a'. Find the approximate potential (upto the dipole term) at a far away point from the square due to this configuration.

$$\nabla \times \mathbf{V}(s,\theta,z) = \left[\frac{1}{s}\frac{\partial V_z}{\partial \theta} - \frac{\partial V_\theta}{\partial z}\right] \mathbf{\hat{s}} + \left[\frac{\partial V_s}{\partial z} - \frac{\partial V_z}{\partial s}\right] \hat{\theta} + \frac{1}{s} \left[\frac{\partial (sV_\theta)}{\partial s} - \frac{\partial V_s}{\partial \theta}\right] \hat{\phi}$$

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