INDIAN INSTITUTE OF SPACE SCIENCE & TECHNOLOGY

B. Tech(I Year) **Physics - II (PH121)** **End Sem.**

5 May 2017 **Duration:3 Hrs** **Full Marks: 100**

Notation : Cartesian coordinates — $\{x, y, z\}$, unit vectors —$\{\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}\}$,
Spherical polar coordinates — $\{r, \theta, \phi\}$, unit vectors —$\{\hat{\mathbf{r}}, \hat{\theta}, \hat{\phi}\}$,
Cylindrical coordinates — $\{s, \phi, z\}$, unit vectors —$\{\hat{\mathbf{s}}, \hat{\phi}, \hat{\mathbf{k}}\}$

Answer all questions

Part A — Each question carries 6 marks

1. Any vector field can be written as a sum of two parts — a gradient of a scalar field and a curl of some vector field: $\mathbf{V}(\mathbf{r}) = \nabla U + \nabla \times \mathbf{A}$. Find U and $\mathbf{A}$ for the following vector fields:

   a) $\mathbf{V} = y(z+1)\hat{\mathbf{i}} + x(z-1)\hat{\mathbf{j}} + yx\hat{\mathbf{k}}$.

   b) $\mathbf{V} = \frac{k}{s}(\hat{\mathbf{s}} + \hat{\phi})$, where k is a constant.

2. A charge Q is uniformly distributed around a thin ring of radius b in the $x-y$ plane with its center at the origin. Find the point on the positive z axis where the electric field is the strongest.

3. A sphere of radius a is charged with uniform density ρ. A spherical cavity of radius $a/2$ is then carved out as shown in the figure. Find the direction and magnitude of the electric fields at the points A, B and C.

4. Two concentric spheres of radius a and b $(b > a)$ carry charges Q and $-Q$, respectively.

If the charges are distributed uniformly over the two spheres, find

a) The work done in assembling the charges

b) The energy stored in the electric fields.

5. A cylindrical slab possesses a electric polarization $\mathbf{P}(\mathbf{r}) = p_0 s\hat{\mathbf{s}}$. Find the bound charge densities ρ_b and σ_b inside the slab and on the surface. s is the radial distance from the axis of the cylinder.

6. A point charge q is placed inside a spherical shell of inner radius a and outer radius b. Assuming the material of the shell to be homogeneous,

   a) Find the displacement field $\mathbf{D}$ and the electric field $\mathbf{E}$ everywhere.

   b) Find the bound charge densities ρ_b and σ_b inside the shell and on its inner and outer surfaces.

7. A current I runs down the y axis from infinity upto the origin, and the flows down along the x axis (to infinity). Find the magnetic field in the region $x > 0, \quad y > 0$.

8. Calculate the self inductance of a cylindrical solenoid 10 cm in diameter and 2 m long. It has a single layer winding containing a total of 1200 turns. Assume the magnetic field inside to be uniform. Will the actual value of self inductance be lower or greater than the calculated L, in view of this approximation?

9. A very long solenoid of radius a, and n turns per unit length, carries a current I. A circular loop of wire of radius $b >> a$ and resistance R is placed coaxial to the solenoid. Suppose the current in the solenoid is varied as $I(t) = I_0 e^{-kt}$, where k is a positive constant.

   a) Find the current induced in the loop of wire, and the total power delivered to the loop.

   b) Find the induced electric field everywhere.

10. A current I flows down a long straight wire of radius a. If the wire is made of linear material with susceptibility χ_m, and the current is distributed uniformly, find the magnetic field a distance s from the axis. Find the bound currents $\mathbf{J}_b$ and $\mathbf{K}_b$, and the net bound current flowing down the wire.

Part B — Each question carries 10 marks

11. A parallel-plate capacitor is made of two large circular plates. The voltage across the plates has the time dependence $V = Vo\cos\omega t$. (You may neglect the fringing fields at the edges).

    a) Use Maxwell's equations and symmetry arguments to determine the electric and magnetic fields in between the plates as functions of time.

    b) Assume d be the distance between the plates. What current flows in the lead wires and what is the current density in the plates as a function of time?

    c) Let σ be the surface charge density of the upper plate and let a be the radius of the plates. Using boundary conditions calculate the magnetic field in the region above the upper plate

12. Which of these are plausible E and B pair ($\omega = ck$, where $c = 1/\sqrt{\mu_0\epsilon_0}$):

    a)$\mathbf{E} = E_0\cos(\omega t - kx)\hat{\mathbf{j}}, \quad \mathbf{B} = \frac{E_0}{c}\cos(\omega t - kx)\hat{\mathbf{i}}$.

    b)$\mathbf{E} = E_0\sin(\omega t)\ln(a/s)\hat{\mathbf{z}}, \ \ \mathbf{B} = \frac{E_0}{\omega s}\cos(\omega t)\hat{\phi}, \ 0 \le s \le a$.

    c)$\mathbf{E} = E_0\frac{\sin\theta}{r}\cos(\omega(t - r/c))\hat{\theta}, \ \mathbf{B} = \frac{E_0}{c}\frac{\sin\theta}{r}\cos(\omega(t - r/c))\hat{\phi}$

13. a) The electric field of a static distribution of charges can be written in terms of the scalar potential as $\mathbf{E} = \nabla V$. However, in a general situation, the Faraday's law negates this possibility. How would you express the Electric field in terms of V and the vector potential $\mathbf{A}$ to circumvent this.

    b) A long straight wire carries a current $I(t) = I_0\cos\omega t$. Find the magnetic field $\mathbf{B}$, electric field $\mathbf{E}$ and the scalar potential V.

14. A very long solenoid of n turns per unit length and radius R carries a current which increases uniformly with time as $I = kt$, where k is a constant.

    a) Calculate the magnetic field inside the solenoid at time t

    b) Calculate the electric field inside the solenoid.

    Consider a cylindrical portion in the solenoid of length l and radius equal to that of the solenoid and coaxial with the solenoid.

    c) Find the rate at which energy flows into the volume enclosed by this cylinder.

    d) Express the energy flow in terms of self-inductance per unit length L.