Dimensionless Numbers in Fluid Mechanics and Heat Transfer

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Nondimensionalization of the governing equations of fluid flow and heat transfer is important for both theoretical and computational reasons. Nondimensional scaling provides a method for developing dimensionless groups that can provide physical insight into the importance of various terms in the system of governing equations. Computationally, dimensionless forms have the added benefit of providing numerical scaling of the system discrete equations, thus providing a physically linked technique for improving the ill-conditioning of the system of equations. Moreover, dimensionless forms also allow us to present the solution in a compact form. Some of the important dimensionless numbers used in fluid mechanics and heat transfer are given below.

- Archimedes Number:
$$Ar = \frac{gL^3\rho(\rho_s - \rho)}{\mu^2} = \frac{Re^2}{Fr}$$

- Atwood Number:
$$A = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$$
Atwood Number is useful in the study of density stratified flows.

- Biot Number:
$$Bi = \frac{hL}{k_s} = \frac{\text{conductive resistance in solid}}{\text{convective resistance in thermal boundary layer}}$$

- Bond Number:
$$Bo = \frac{\rho g L^2}{\sigma} = \frac{We}{Fr}$$

- Brinkman Number:
$$Br = \frac{\mu U^2}{k(T_w - T_o)} = PrEc$$
Brinkman number is related to heat conduction from a wall to a flowing viscous fluid. It is commonly used in polymer processing. If $Br \ll 1$, the viscous dissipation can be neglected.

- Capillary Number:
$$Ca = \frac{\mu U}{\sigma} = \frac{We}{Re}$$

- Cauchy Number:

$$Ca = \frac{\rho U^2}{K} = M^2$$

- Centrifuge Number:

$$Ce = \frac{\rho \Omega^2 L^3}{\sigma} = \frac{We}{Ro^2}$$

- Dean Number:

$$De = \frac{Re}{\sqrt{R/h}}$$

Dean number deals with the stability of two-dimensional flows in a curved channel with mean radius R and width $2h$.

- Deborah Number:

$$De = \frac{\tau}{t_p} = \frac{\text{relaxation time}}{\text{characteristic time scale}}$$

Deborah Number commonly used in rheology to characterize how "fluid" a material is. The smaller the De, the more the fluid the material appears.

- Eckert Number:

$$Ec = \frac{U^2}{c_p \Delta T} = \frac{\text{dynamic temperature induced by fluid}}{\text{characteristics temperature difference in the fluid}}$$

Eckert number represents the kinetic energy of the flow relative to the boundary layer enthalpy difference. Ec plays an important role in high speed flows for which viscous dissipation is significant.

- Ekman Number:

$$Ek = \frac{\mu}{\rho \sigma L^2} = \frac{\text{viscous force}}{\text{Coriolis force}}$$

- Eötvös Number:

$$Eo = \frac{\Delta \rho g L^2}{\sigma} = \frac{We}{Fr}$$

- Euler Number:

$$Eu = \frac{\Delta p}{\rho U^2} = \frac{\text{pressure force}}{\text{inertial force}}$$

- Fourier Number:

$$Fr = \frac{\alpha t}{L^2} = \frac{\text{rate of heat conduction}}{\text{rate of thermal energy stored}}$$

Fourier number represents the dimensionless time. It may be interpreted as the ratio of current time to time to reach steady-state.

- Froude Number:

$$Fr = \frac{U^2}{gL} = \frac{\text{inertial force}}{\text{gravitational force}}$$

- Galileo Number:

$$Ga = \frac{\rho^2 g L^3}{\mu^2} = \frac{Re^2}{Fr}$$

- Graetz Number:

$$Gz = \frac{U d_h}{\nu} = \frac{d_h Pe}{L}$$

- Grashof Number:

$$Gr = \frac{g\beta(T_{\text{hot}} - T_\infty)L^3}{\nu^2} = \frac{\text{buoyancy force}}{\text{viscous force}}$$

- Hagen Number:

$$Hg = -\frac{dp}{dx}\frac{\rho L^3}{\mu^2} =$$

  Hagen Number is the forced flow equivalent of Grashof number.

- Jakob Number:

$$Ja = \frac{c_p(T_w - T_{\text{sat}})}{h_{fg}} =$$

  Jakob number represents the ratio of sensible heat to latent heat absorbed (or released) during the phase change process.

- Knudsen Number:

$$Kn = \frac{\lambda}{L} = \frac{\text{length of mean free path}}{\text{characteristic length}}$$

- Laplace Number:

$$La = \frac{\rho\sigma L}{\mu^2} = \frac{Re^2}{We}$$

- Lewis Number:

$$Le = \frac{\alpha}{D_{AB}} = \frac{\text{thermal diffusivity}}{\text{mass diffusivity}}$$

- Mach Number:

$$M = \frac{U}{a} = \frac{\text{inertia force}}{\text{elastic (compressibility) force}}$$

- Marangoni Number:

$$Ma = -\frac{d\sigma}{dT}\frac{L\Delta T}{\mu\alpha}$$

  Marangoni number is the ratio of thermal surface tension force to the viscous force.

- Morton Number:

$$Mo = \frac{g\mu^4}{\Delta\rho\sigma^3} = \frac{We^2}{FrRe^4}$$

- Nusselt Number:

$$Na = \frac{hL}{k_f}$$

Nusselt number represents the dimensionless temperature gradient at the solid surface.

- Ohnesorge Number:

$$Oh = \frac{\mu}{\sqrt{\rho \sigma L}} = \frac{\sqrt{We}}{Re}$$

- Peclet Number:

$$Pe = \frac{UL}{\alpha} = RePr = \frac{\text{inertia (convection)}}{\text{diffusion}}$$

- Prandtl Number:

$$Pr = \frac{\nu}{\alpha} = \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}}$$

- Rayleigh Number:

$$Ra = \frac{g\beta(T_{\text{hot}} - T_\infty)L^3}{\nu\alpha} = GrPr = \frac{\text{buoyancy}}{\text{viscous} \times \text{rate of heat diffusion}}$$

- Reynolds Number:

$$Re = \frac{\rho UL}{\mu} = \frac{\text{inertial force}}{\text{viscous force}}$$

- Richardson Number:

$$Ri = \frac{g\beta(T_{\text{hot}} - T_\infty)L}{U^2} = \frac{Gr}{Re^2} = \frac{\text{buoyancy force}}{\text{inertial force}}$$

- Rossby Number:

$$Ro = \frac{U}{\Omega L} = \frac{\text{inertial force}}{\text{Coriolis force}}$$

- Rotating Froude Number:

$$Fr_R = \frac{\Omega^2 L}{g} = \frac{Fr}{Ro^2}$$

- Schmidt Number:

$$Sc = \frac{\nu}{D_{AB}} = LePr = \frac{\text{momentum diffusivity}}{\text{mass diffusivity}}$$

- Sherwood Number:

$$Sh = \frac{h_m L}{D_{AB}}$$

Sherwood number represents the dimensionless concentration gradient at the solid surface.

- Stanton Number:

$$St = \frac{h}{\rho U c_p} = \frac{Nu}{RePr}$$

Stanton number is the modified Nusselt number. It is used in analogy between heat transfer and viscous transport in boundary layers.

- Stefan Number:

$$St = \frac{c_p dT}{L_m} = \frac{\text{specific heat}}{\text{latent heat}}$$

Stefan number is useful in the study of heat transfer during phase change.

- Stokes Number:

$$Stk = \frac{\tau U_o}{d_c} = \frac{\text{stopping distance of a particle}}{\text{characteristic dimension of the obstacle}}$$

Stokes number is commonly used in particles suspended in fluid. For $Stk \ll 1$, the particle negotiates the obstacle. For $Stk \gg 1$, the particle travels in straight line and eventually collides with obstacle.

- Strouhal Number (for oscillatory flow) Number:

$$St = \frac{L}{U t_{\text{ref}}} = \frac{\text{inertia (local)}}{\text{inertia (convection)}}$$

If t_{ref} is taken as the reciprocal of the circular frequency ω of the system, then

$$St = \frac{\omega L}{U}$$

- Taylor Number:

$$Ta = \frac{\rho^2 \Omega_i^2 L^4}{\mu^2}$$

where $L = \left[r_i (r_o - r_i)^3\right]^{1/4}$

- Weber Number:

$$We = \frac{\rho U^2 L}{\sigma} = \frac{\text{inertial force}}{\text{surface tension force}}$$

- Womersley Number:

$$\alpha = L\sqrt{\frac{\rho \omega}{\mu}} = \sqrt{\pi ReSt}$$

Womersley number is used in biofluid mechanics. It is a dimensionless expression of the pulsatile flow frequency in relation to the viscous effects.

Nomenclature:

a $\rightarrow$ speed of sound
c_p $\rightarrow$ specific heat at constant pressure
D_{AB} $\rightarrow$ mass diffusivity coefficient
dT $\rightarrow$ temperature difference between phases
d_c $\rightarrow$ characteristic dimension of the obstacle
d_h $\rightarrow$ hydraulic diameter of the duct
g $\rightarrow$ gravitational acceleration
h $\rightarrow$ heat transfer coefficient
h $\rightarrow$ width of the channel
h_{fg} $\rightarrow$ latent heat of condensation
h_m $\rightarrow$ mass transfer coefficient
K $\rightarrow$ bulk modulus of elasticity
k $\rightarrow$ thermal conductivity of fluid
k_f $\rightarrow$ thermal conductivity of fluid
k_s $\rightarrow$ thermal conductivity of solid
L $\rightarrow$ characteristic length scale
L_m $\rightarrow$ latent heat of melting
R $\rightarrow$ radius of the channel
r_i $\rightarrow$ radius of the inner cylinder
r_o $\rightarrow$ radius of the outer cylinder
T_{hot} $\rightarrow$ temperature of the hot wall
T_{ref} $\rightarrow$ reference temperature
T_o $\rightarrow$ bulk fluid temperature
T_{sat} $\rightarrow$ saturation temperature
T_w $\rightarrow$ wall temperature
T_∞ $\rightarrow$ quiescent temperature of the fluid
t $\rightarrow$ time
t_{ref} $\rightarrow$ reference time
t_p $\rightarrow$ characteristic time scale
U $\rightarrow$ characteristic velocity scale
U_o $\rightarrow$ fluid velocity far away from the object
dp/dx $\rightarrow$ pressure gradient
$d\sigma/dT$ $\rightarrow$ rate of change of surface tension with temperature
α $\rightarrow$ thermal diffusivity of fluid
β $\rightarrow$ volumetric thermal expansion coefficient
Δp $\rightarrow$ characteristic pressure difference of flow
ΔT $\rightarrow$ characteristic temperature difference
$\Delta \rho$ $\rightarrow$ difference in density of the two phases
λ $\rightarrow$ mean free path distance
μ $\rightarrow$ viscosity of fluid
ν $\rightarrow$ kinematic viscosity of fluid

ρ $\rightarrow$ density of fluid
ρ_1 $\rightarrow$ density of heavier fluid
ρ_2 $\rightarrow$ density of lighter fluid
ρ_s $\rightarrow$ density of solid
σ $\rightarrow$ surface tension
τ $\rightarrow$ relaxation time
Ω $\rightarrow$ angular velocity
ω $\rightarrow$ circular frequency
ω_i $\rightarrow$ angular velocity of inner cylinder