

The Gram-Schmidt orthogonalization process for computing an orthonormal basis  $B_{ON} = \{w_1, w_2, \dots, w_n\}$  for a finite dimensional inner product space  $V$  with basis  $B = \{u_1, u_2, \dots, u_n\}$  is as follows:

Step 1 : Let  $v_1 = u_1$

Step 2 : Compute  $v_2, v_3, \dots, v_n$  by the formula :

$$v_2 = u_2 - \text{Proj}_{v_1}(u_2)$$

$$v_3 = u_3 - \text{Proj}_{v_2}(u_3) - \text{Proj}_{v_1}(u_3)$$

and so on.

The set  $\{v_1, v_2, \dots, v_n\}$  is an orthogonal basis for  $V$ .

Step 3 : Let  $w_i = \frac{v_i}{\|v_i\|}$ .

Then,  $B_{ON} = \{w_1, w_2, \dots, w_n\}$  is an orthonormal basis for  $V$ .