Experimental and Numerical Investigations on the Dynamics of Highly Flexible Cantilever Beams

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by

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Abstract

This work details the dynamic characteristics of thin, long, and highly flexible cantilever beams of different aspect ratios (length-to-thickness ratio). The thin, long, and highly flexible cantilever beams can undergo very large deflections within the elastic limit. The rigid body rotations play a significant role in describing deformations rather than the elastic deformation part. The classical finite element formulation with geometric nonlinearity fails to model the behavior accurately due to the assumption of negligible rigid body rotations. Two cantilever beams are considered, one with an aspect ratio of 643.92:1 and one of 1768:1. An Absolute Nodal Coordinate Formulation (ANCF), a flexible multibody dynamic modeling approach, is attempted here to model large deflections accurately, including rigid body rotations. A 3D gradient-deficient ANCF beam element with bending characterized using curvature terms is used for modeling the beam, and the approach is validated using various load cases and with different damping models. The semi-analytical solution approaches are used to evaluate the steady-state dynamics of the system without going into transients. The Time Variational Method (TVM), a semi-analytical solution in the time domain, is used for the harmonic response evaluation of thin cantilever beams of higher aspect ratios modeled with ANCF. This solution methodology evaluated the linear and nonlinear harmonic responses. The harmonic response experiments using an electrodynamic shaker agreed well with the computations. Furthermore, to understand the nonlinear free vibration characteristics of the thin cantilever beams, a solution technique using TVM coupled with ANCF is proposed to evaluate the nonlinear normal modes (NNM). The NNMs are extracted experimentally using the phase resonance approach, which agrees well with the computations. It is observed that ANCF with TVM can predict additional internal resonances, which finite element formulation could not predict. Also, the softening behavior of the higher modes of the cantilever beam is present in both computations and experiments. An energy-balance approach is used to study the NNM characteristics of an autonomous continuous system using energy as an input parameter. The numerical integration results are used to validate the solution methodology.