CONTROLLABILITY AND OBSERVABILITY OF NETWORKED SYSTEMS

A thesis submitted

in partial fulfillment for the award of the degree of

Doctor of Philosophy

in

Mathematics

by

ABHIJITH AJAYAKUMAR

supervised by

PROF. RAJU K. GEORGE

Department of Mathematics

Indian Institute of Space Science and Technology

Thiruvananthapuram, India

August 2024

Abstract

'Controllability' is a fundamental feature of dynamical systems introduced by R.E. Kalman in the 1960s. Various notions of controllability, such as state controllability, structural controllability, and so on, are proposed in the literature, and controllability conditions for both linear and non-linear systems were obtained by many authors. State controllability deals with the ability of the system to steer itself from an arbitrary initial state to a desired final state using suitable control functions, whereas structural controllability, introduced by C.T. Lin aims at setting some values to the non-zero parameters in the system matrices so that the resultant system is state controllable in the sense of Kalman. Another important feature of a control system introduced by Kalman is Observability, which focusses on the ability of reconstructing the internal states of the system from the knowledge of its outputs during an interval. Over the last few decades, investigations on controllability and observability of dynamical systems have drawn the interest of many scholars, who have made tremendous progress and acquired many new insights. Majority of these discoveries pertain to single higher-dimensional control systems. However, the prevalence of networked control systems in the actual world is far higher than that of single stand-alone control systems. In general, modelling complex systems necessitates the use of a group of separate systems linked together via an interconnection structure. The controllability and observability of large-scale complex networked systems presents fascinating research problems. There is a lot of interest in the study of controllability and observability of networked systems, since it has applications in many different scientific and technological disciplines. These studies include a range of system characteristics, including structural complexity, node dynamics and interactions between distinct nodes. Despite significant research in this area, there is no general result about the controllability and observability of networked systems in the literature that shows how the intrinsic features of the network and the dynamics of the individual nodes affect the controllability and observability of the networked system. The majority of the available results in the literature are for networked systems having identical individual nodes. However, in practice, not all individual nodes may possess the same dynamics. The objective of this thesis is to investigate the controllability and observability of networked systems having non-identical individual nodes with a focus on the effects of individual node dynamics and network topology. The obtained theoretical results are substantiated with numerical examples.