Inverse Source Problems for the Damped Euler-Bernoulli Beam and Kirchhoff Plate Equations

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by

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Abstract

The direct and inverse problems for the Euler-Bernoulli beam and Kirchhoff-Love plate models have been extensively studied over the years, and they continue to be an active area of research due to their applications in science and engineering. In this aspect, we mainly focus on the inverse source problems of the Euler-Bernoulli beam and Kirchhoff-Love plate equations with various damping mechanisms. More precisely, the research reported in the thesis mainly deals with the inverse problems of identifying unknown source terms in the Euler-Bernoulli beam with viscous and Kelvin-Voigt dampings, rectangular Kirchhoff-Love plate with viscous damping, thermoelastic plate with structural damping.

As explained in the introduction of the thesis, the unique determination of a spatial load in the undamped beam equation from final time measurement is not a feasible problem. We study the effect of viscous damping in the unique determination of unknown spatial load in a simply supported Euler-Bernoulli beam from measured final time displacement. By considering two specific temporal loads, we obtain sufficient conditions on the damping parameter and admissible final time interval to uniquely express the spatial load in terms of an infinite series using the Singular Value Decomposition (SVD) method. Next, we discuss the inverse problem of determining the unknown transverse shear force (boundary data) in the Euler-Bernoulli beam in the presence of the Kelvin-Voigt damping from measured deflection and bending moment. The inverse boundary value problem of determining the shear force acting on the inaccessible tip of the microcantilever, one of the key components of Transverse Dynamic Force Microscopy (TDFM), is important for understanding biological specimen images at submolecular precision. The considered inverse problems are transformed into minimization problems for Tikhonov functionals and show that the regularized functionals admit a unique solution for the inverse problems. In this work, we also prove remarkable Lipschitz stability estimates for the transverse shear force in terms of the given measurement by a feasible condition only on the Kelvin-Voigt damping coefficient using the variational methods. The required solvability of direct and adjoint problems is obtained under the minimal regularity of the admissible shear force, which turns out to be the regularizing effect of the Kelvin–Voigt damping.

The analysis of the inverse source problem of the beam is further explored for the unique reconstruction of spatial load and the stability of reconstructing the spatial load

in the Kirchhoff-Love plate in the presence of viscous damping using the regularization technique and spectral method. In this study, the inverse problem is first posed as a minimization problem of a regularized Tikhonov functional and obtained a unique solution to the minimization problem. We established a stability estimate under feasible conditions on final time and damping parameter. Then, the same inverse source problem is studied by the SVD method, and we concluded that the solution obtained by these two methods is equivalent. Besides, with the help of singular values of the input-output operator and regularity assumption on temporal load, we derived stability estimates for the regularized and SVD solutions of the inverse problem.

In the final work, we further extend the study for the inverse problem of simultaneously identifying the mechanical load and heat source in a structurally damped thermoelastic system describing a homogeneous and elastically and thermally isotropic plate from the vertical displacement measured at the final time. The inverse problem is reformulated as a minimization problem for the Tikhonov functional using the Tikhonov regularization method. We prove that the regularized Tikhonov functional admits a unique solution in the naturally defined set of admissible sources. An upper bound for the final time is established to derive a stability estimate for the inverse problem by invoking a first-order necessary optimality condition for the minimization problem. This stability result also gives rise to the uniqueness of the solution to the inverse problem. The results established in this work help to analyze the influence of thermal and mechanical loading that results in materials deflection, which, in turn, is vital for material science and engineering applications.