A Theoretical Framework for Sparse Recovery Algorithm Design and Evaluation in Compressed Sensing Perspective

A thesis submitted
in partial fulfillment for the award of the degree of

Doctor of Philosophy

by

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October 2023

Abstract

Compressed sensing is a well established signal acquisition method. This involves sampling of correlated and integrated signal at reduced sampling rate. The compressed sensed signals are not direct time domain representations, hence the reconstruction of the original signal involve function minimization methods or matrix minimization methods. Large numbers of sparse signal reconstruction algorithms are developed in the recent time. The availability of large numbers of reconstruction algorithms create dilemma in choosing a particular method for a specific reconstruction application. The recovery algorithms are generally compared in terms of computational complexity, computational time, probability of recovery and recovery precision. Typically absolute Mean Squared Error (MSE) and relative MSE are used to compare the recovery precision of various sparse recovery algorithms. However, these two metric alone are not sufficient to assess all algorithms. The research work presented in this thesis starts with a novel algorithm evaluation strategy by ranking the algorithms based on the observable similarity between the original and the reconstructed signal.

The thesis presents four consequential developments: first is the description of a novel method for analysis and ranking of sparse recovery algorithms. Second a frame work for improving the accuracy of sparse signal recovery algorithms using iterative residue estimation, proximal projection and segmented thresholding and the development of two new recovery methods using the proposed frame work. Third, the evaluation of an IoT based computing platform for the implementation of the proposed sparse reconstruction algorithms. Fourth the implementation of the proposed algorithm in real-time networked data acquisition scenario.

The signal reconstruction from the compressed sensed data need iterative methods since the sparse measurement matrix is analytically non invertible. The iterative thresholding and ℓ_0 function minimization are of special interest as these two operations provide sparse solution. However these methods need an inverse operation

corresponding to the measurement matrix for estimating the reconstruction error. The pseudo-inverse of the measurement matrix is used in general for this purpose. In the second part of the work, a sparse signal recovery framework using an approximate inverse matrix $\mathbf{Q}$ and iterative segment thresholding of ℓ_0 and ℓ_1 norm with residue addition is presented. Two recovery algorithms are developed using this framework. The ℓ_0 based method is later developed into a basis function dictionary based network for sparse signal recovery. The proposed framework enables the user to experiment with different inverse matrices to achieve better sparse signal recovery efficiency and implement the algorithm in computationally efficient way. Also, the proposed framework is used in the development of a cascade computing network for sparse signal recovery.

In the third part, the functional evaluation of an IoT based computing platform for implementation of the proposed sparse reconstruction algorithm is presented. The Beagleboard is used as prototyping and product development platform, however the full computational and networking capability of its ARM processor AM3358 and programmable real time unit are not fully utilized due to the bandwidth limitations of the networking device used in the board. The network performance evaluation of the board is performed experimentally and compared with the real time requirements of a networked commanding and data acquisition system. The feasibility of using this board for real time applications with a response latency of $< 20ms$ is studied. The observations from the timing analysis indicate that the timing constraints need to be implemented on the system for getting real-time performance.

The work presented in this thesis concludes by implementation of the proposed sparse recovery algorithm on the IoT computing platform, for realization of a networked system for acquisition and reconstruction of naturally sparse events. The naturally sparse event acquired here is the surge in ground potential. A common reason for electronic measurement anomaly is the inadvertent rise in ground potential with respect to measurement ground. The ground potential rise happens during current leakage to the ground from lightning or from power grid and leads to catastrophic failure unless appropriate preventive action is taken to isolate the sensitive measurement systems. A networked system for compressed sensing and transmission of the ground potential measurement values to a remote data monitoring station is demonstrated using the proposed method and platform. The limited processing power of such devices is not sufficient enough to run high computation intensive routing algorithms. Hence a lightweight routing algorithm for this purpose is also proposed. The discussion on the reliability of such systems is presented for completeness. The multipath route discovery strategy presented here reconfigures the network to an optimal configuration with respect to energy dissipation and node distribution.

In brief, the work presented in this thesis begins with analysis of various sparse signal reconstruction algorithms, then proposes a novel metric for ranking these algorithms using the signal similarity and probability. Based on the salient features of various sparse signal reconstruction algorithm, a framework for improving the performance of these algorithm is presented. This framework is used in the development of a function dictionary based computing network for sparse signal reconstruction. To implement the proposed algorithms, an IoT based computing platform is selected and evaluated to confirm that it meets the computational requirements. A distributed data acquisition system for measurement and reconstruction of sparse signal using the proposed algorithm is presented. Additionally a low power data routing algorithm for the IoT based system is also developed to support the data communication.