Lower Dimensional Approximation of Some Problems for Thin Piezoelectric and Elastic Shells with Nonuniform Thickness

A thesis submitted

in partial fulfillment for the award of the degree of

Doctor of Philosophy

by

Job Mathai

Department of Mathematics

Indian Institute of Space Science and Technology

Thiruvananthapuram, India

July 2021

Abstract

Elastic or piezoelectric structures are three dimensional structres. They are very much used in various applications(eg: aerospace, biomechanics etc). Often when the thickness of the elastic or piezoelectric structure is "very small" when compare to other dimensions lower dimensional models are preferred to the actual three dimensional model.

Lower dimensional approximation of elastic and piezoelectric plates and shells with "uniform thickness" have been extensively studied. In this thesis we study the corresponding problems for "non-uniform thickness". More precisely, we study the two dimensional approximation of eigenvalue problem for piezoelectric shallow shells and flexural shells with non-uniform thickness and dynamic problem for elastic shallow shells with non-uniform thickness. We show that the solution of the three dimensional problem converge to the solutions of two dimensional model when the thickness of the shell (denoted by ϵ) goes to zero.

In the second chapter we consider eigenvalue problem for thin piezoelectric shallow shells (i.e, the curvature goes to zero as the thickness of the shell goes to zero) with non-uniform thickness. The technique used here for proving convergence rely on those used by J.Raja and N.Sabu [70] for two dimensional approximation of boundary value problem for piezoelectric shallow shells with non-uniform thickness. We first transform the problem to a domain independent of the thickness parameter ϵ and show that the scaled eigenvalues are $o(\epsilon^2)$ and the corresponding scaled eigensolutions converge to the eigensolutions of a two dimensional model. We also show that all the eigensolutions of the two dimensional problem occur this way, i.e, each eigensolution of the two dimensional model is the limit of a sequence of eigensolutions of the three dimensional problem as the thickness of the shell goes to zero.

In the third chapter we consider eigenvalue problem for flexural shells (i.e, the space of inextensional displacement is non zero) with non-uniform thickness. Here also we first transform the problem to a domain independent of ϵ and show that the eigenvalues are $o(\epsilon^2)$ and the corresponding scaled eigensolutions converge to the eigensolutions of a two dimensional model. We also show that all the eigenvalues of the limit problem are limit of sequence of eigensolutions of the three dimensional problem as the thickness of the shell goes to zero.

In the fourth chapter we consider a dynamic problem for elastic shallow shells with non-uniform thickness and we show that under suitable scalings on the applied forces and unknowns the solutions of the three dimensional model converge to the solution of two dimensional model as the thickness of the shell goes to zero.