Some Classes of Probability Distributions Constructed from Phase Type Random Variables

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by

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Abstract

Developing versatile classes of probability distributions that can model a large variety of shapes and offer a high degree of computational tractability is having paramount importance in probability theory, as it helps to study several practical systems appearing across various disciplines. In line with this idea, here we mainly focus on developing some classes of probability distributions, which are governed by Markov processes, in different domains that form dense subsets of the set of all distributions defined on those domains. In this direction, we start with developing a class of distributions of random variables that are functions of phase type (PH) random variables. Furthermore, this work provides an estimation procedure for finding the maximum likelihood estimates of the parameters of this class and also computes the Fisher Information Matrix (FIM), which is used to construct the confidence regions of the population parameters. The Expectation-Maximization algorithm (EM algorithm) is used for this purpose.

A new class of circular distributions called Wrapped Bilateral Phase Type (WBPH) class of distributions is derived by using the wrapping method. The denseness of this new class in the set of all circular distributions is proved and some of its major characteristics are established. To illustrate its practical applications, this class is used to model the direction of the wind in some regions. Also, by using the same concept, a new class of linear distributions, with support on any real interval of a finite length, which is dense in the set of all distributions with support on that interval is derived.

As another work, a multivariate class of distributions with support I, a $k-$orthotope in $[0, \infty)^k$, which is dense in the set of all $k-$dimensional distributions with support I is introduced. Some properties of this new class of distributions, named Multivariate Finite Support Phase Type Distributions (MFSPH), are studied. Estimates of the parameters of a particular class of Bivariate Finite Support Phase Type (BFSPH) Distributions are found by using the EM algorithm as well as the Direct Method (DM). The FIM is also calculated using DM. Some applications of BFSPH distributions are also discussed.