On the dynamics of a periodically driven spheroid in a variety of fluid flows at low Reynolds numbers

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Abstract

The purpose of this thesis is to investigate the orientation dynamics and resonance properties of oscillations of a periodically driven non-spherical micro-spheroid in a set of Newtonian unsteady viscous fluids at low Reynolds numbers. We neglect the particle-particle interactions by assuming a sufficiently diluted suspension. As a first case, we derive and investigate a perturbation solution of motion of the harmonically-forced rectilinear displacement of a weak eccentric spheroid along its axis of symmetry in the presence and absence of memory forces at resonance. The dependence of the body's aspect ratio, free oscillation frequency, and particle–fluid density ratio on the motion solutions is examined and analyzed. A governing equation inclusive of the effect of damping, Basset memory, and second history integral forces at small Reynolds numbers is derived, and then we proceed to obtain an analytical solution of this equation at resonance. Expressions of the conventional Q-curves, amplitude-frequency, and phase-frequency oscillations of the spheroid with the natural frequency are also derived.

Considering the applications of the orientation of particles in three-dimensional systems, we study the dynamics of a rigid particle in a quiescent fluid, uniform flow, and oscillating flow at low Reynolds numbers. We have derived the system of differential equations that describe the motion of an arbitrarily forced spheroid in each flow at a low Reynolds number. These governing equations are non-linear and contain a history term of all the past positions and velocity. Therefore, obtaining their analytical solution is non-trivial, and suitable numerical methods are employed to study the spheroid transport. The novel features of this study

include periodic forces in an arbitrary direction, The hydrodynamic forces arising due to the disturbance of the velocity fluctuations the forces induced due to the non-spherical nature of the rigid body. Next, we consider a periodically forced prolate spheroid suspended in an oscillating Newtonian fluid in the low-Reynolds number limit. We study the characteristics of the solutions of the particle due to the periodic force applied on the spheroid particle and induced hydrodynamic force acting on the particle. We obtain the governing equations of the proposed problem by using an appropriate expression for the hydrodynamic force. We also examine the orientation profile of a rigid body suspension in a time-dependent uniform flow at low Reynolds numbers under the action of an external periodic field.

In summarizing the results and the applications, we see that the problems considered here have significant contributions from fundamental and technological aspects. This proposed work help in understanding the role of aspect ratio, density ratio, and free frequency on the oscillation properties of the particle. The observed phenomena may give new insights into physics, especially regarding the quantum of velocity disturbances due to particle shape. This study can be used to analyze the oscillation variations of particles having arbitrary eccentricity in the presence of history integral terms and/or other external forces like a magnetic force, acoustic radiation force, electric force, etc. Technically, we can use the dependencies of properties on the controllable parameters for devising better particle separation for characterizing suspensions having desired properties. The analytical solutions obtained at resonance might be important in testing software designed for more complicated and realistic systems, hence striking a good balance between complication and tractability. The solutions may have practical applications in experiments involving more complex systems, mainly to understand the effect of acoustic waves on micro-particle transport. The work can be extended further in many directions, especially considering the effect of Brownian motion and particle-particle interactions.