Lowest Order Virtual Element Approximations for Unsteady Fluid Flow Problems

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Abstract

Problems in continuum mechanics are a constant source of systems of partial differential equations (PDEs) which are often difficult to solve. Among contemporary numerical methods designed for these types of problems, virtual element methods (VEMs) constitute a recent family of discretization schemes constructed using polytopal meshes, that are proven to be robust under many different scenarios. In this thesis, we focus on the developments of VEMs for the approximation of certain types of non-stationary coupled fluid flow problems. More precisely, the type of equations that are considered herein includes transient Stokes, Navier-Stokes, Biot, and coupled advection-diffusion-reaction and poroelasticity equations, the latter system describing species interaction within fully saturated deformable porous media. Using classical regularity assumptions on the solutions to the continuous set of governing equations, we construct lowest-order virtual element discretizations for each of these problems. An appealing feature of the resulting schemes is that the discrete velocities are locally divergence-free for incompressible flow problems and that the constructed virtual element spaces satisfy the necessary inf-sup conditions which permit to establish unique solvability of the associated discrete problems and Céa estimates for the approximate solutions. For the time discretization, a classical backward Euler scheme is employed, and we rigorously derive the main properties of the semi- and fully-discrete schemes for all problems. Moreover, by introducing appropriately defined projection operators, optimal *a priori* error estimates are established in natural norms for all field variables that are natural unknowns in the specific formulation. Further, for each problem, several numerical experiments are presented. They serve to illustrate the performance of the proposed schemes and also to validate experimentally the theoretical rates of convergence predicted by the error analysis.