Fractional Order Controllers for Complex Valued Systems and System with Multiple Nonlinearities

A thesis submitted

in partial fulfillment for the award of the degree of

Doctor of Philosophy

by

P. Sathishkumar

Department of Avionics

Indian Institute of Space Science and Technology

Thiruvananthapuram, India

November 2019

Abstract

Fractional Calculus (FC) is a branch of mathematics which generalises classical integer-order calculus to handle integrals and derivatives of *arbitrary* orders. Recently, the FC has received attention in various science and engineering fields including control theory. In control theory, one deals with the modelling, design and analysis of Fractional Order (FO) systems and controllers, whose dynamics are governed by FO differential equations. In this thesis, unified tuning expressions of Fractional Order Controllers (FOCs) for the proposed universal plant structure to meet Wang et al specifications are derived. Further, this work is extended to tune the parameters of the Complex Coefficient Integer Order Controllers (CCIOCs) and Complex Coefficient Fractional Complex Order Controllers (CCFCOCs) for such universal plant. This thesis also investigates the limit cycle prediction using various methods and its suppression using FOCs for system with multiple nonlinearities.

The thesis start with the proposal of defining plant model having complex coefficients and complex order derivatives plus dead time as a universal plant structure. Then, unified tuning expressions of FOCs such as PI^{α}, $[PI]^{\alpha}$, PD^{β}, $[PD]^{\beta}$ and $K_c\left(\frac{s}{\omega_{gc}}\right)^{\alpha+j\beta}$ are derived to meet the desired gain crossover frequency, phase margin and Isodamping property (Wang et al specifications) for the proposed universal plant structure using its positive frequency (ω^{+}) information. Two different case studies are simulated to demonstrate the controller tuning for the proposed structure. Performing the tuning of controllers by considering ω^{+} information is applicable only for Integer Order (IO)/FO plants containing real coefficients which have an even symmetrical magnitude and odd symmetrical phase behaviour in frequency response.

In general, plant with complex coefficients provides unsymmetrical magnitude and phase behaviour in its frequency response. Hence, tuning of controllers for universal plants by considering only its ω^{+} information alone is not adequate. This type of tuning reduces stability margins and deteriorates its time response. Therefore, tuning of controllers for universal plants require both ω^{+} and negative frequency (ω^{-}) information which in turn demands complex coefficient controllers.

To address this problem, Complex Coefficient Integer Order Controllers (CCIOCs) and Complex Coefficient Fractional Complex Order Controllers (CCFCOCs) are proposed. Unified tuning expressions are derived for CCIOCs by considering both ω^{+} and ω^{-} information of such universal plant. In case of CCFCOCs, the controller parameters are

obtained through optimization technique due to its difficulty in solving by analytical approach. Numerical simulations are performed for few case studies to demonstrate the proposed CCIOCs and CCFCOCs. The results are compared with real coefficient Integer Order Controllers (IOCs) and FOCs tuned only in ω^+.

Then, the focus is on the prediction of limit cycle using graphical approaches and its suppression using FOCs for system containing multiple nonlinearities. In practical systems having separable hard nonlinearities, sustained oscillation is detected at the steady state response due to the presence of stable limit cycles. An optimization problem is proposed to tune IOCs/FOCs parameters for system with multiple nonlinearities to suppress the limit cycle magnitude in addition to meet the desired closed loop specifications. To extend the applicability of the existing Nyquist plot for predicting this limit cycle, an Input Dependent Nyquist Plot (IDNP) is proposed in this work. A servo system with backlash and relay nonlinearities is considered as a case study for limit cycle prediction with obtained FOCs using an IDNP. The predicted limit cycle information is compared with optimization results, Input Dependent Root Locus (IDRL) and are validated through closed loop simulations. It is found that FOCs have remarkably eliminated the limit cycle oscillation in comparison to IO PD controller at steady state of the closed loop response. Further, the robustness of the designed controllers are tested under system parameter uncertainty, disturbance and measurement noise conditions.