STUDIES ON CONTROLLABILITY OF DYNAMICAL SYSTEMS WITH IMPULSES AND TIME-DELAY CONTROLS

A thesis submitted

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by

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Abstract

One of the fundamental properties of the dynamical systems is "controllability", introduced by R. Kalman in 1960s. Since then it has become an active topic of research in the modern control theory. This thesis is devoted to explore the controllability issues for certain classes of finite-dimensional continuous dynamical control systems possessing impulses in state and time-delay in controls. The main reason for considering these types of systems is that, many of the evolution processes which occur in real life, like medicine, biology, computer networking, neural networks, information science, artificial intelligence, telecommunications, robotics etc., are modelled by such systems involving state functions which exhibit an abrupt changes at certain moments of time, that in the form of impulses. Similarly in many systems, like chemical process systems, hydraulically actuated systems, combustion systems, population dynamics etc., the past values of the control function exerts its influence on the present, and hence on the future of the state function, and these phenomena are modelled by the time-delay control systems. Though some research has been conducted on the controllability of certain classes of impulsive and delay systems, but it is not fully explored, especially when it comes to nonlinear systems, networked systems and systems modelled by Lyapunov equations. Further, many of the anomalous processes shows a very complex behaviour which can be studied if their dynamics are modelled by fractional-order differential equations. In case of fractional systems also, like the classical derivative models, the controllability property is not fully examined, in particular for the systems possessing time-delay controls. Based on all the observations, the objectives of this thesis is in the establishment of the controllability properties of the following classes of dynamical control systems:

1. Impulsive systems with time-delay controls.
2. Systems described by the fractional derivatives with delays in control.
3. Systems modelled by the Lyapunov matrix equations with impulses and time-delays in the control function.
4. Finite-dimensional linear systems carrying impulses in state.
5. Networked control systems exhibiting impulses.

We use the tools of linear and nonlinear operator theory such as fixed-point theorems—Schauder's fixed-point theorem, Banach contraction principle—and spectral theory, to obtain the controllability results. Numerical examples are provided to substantiate the theoretical results.