Some aspects of phase estimation in paraxial wave optics

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Abstract

Estimation of phase information through the knowledge of intensity information is a well-studied problem. The intensity measurements can be made, for instance, at two or more transverse planes in the direction of wave field propagation, or using an interferometric setup. The phase is then retrieved from such intensity measurements using several well-known techniques. These retrieved phases could possess dislocations, i.e., phase is undefined at points where the intensity vanishes. Phase retrieval through many plane intensity measurements, in general, requires the knowledge of a general first-order optical system (FOOS). Thus, we first realize any general FOOS using the optical transformations: free propagation, thin convex lens, and thin cylindrical lens of positive focal length.

As is well known, any general FOOS is characterized by a matrix in $Sp(4,\mathbb{R})$, symplectic group in 4 dimensions with its entries being real. Using Euler decomposition, we first write the given $Sp(4,\mathbb{R})$ matrix as a product of orthogonal symplectic matrices in 4 dimensions and differential magnifier matrix in 4 dimensions. These matrices are then realized in terms of free propagation and thin lenses of positive focal length. It is shown that not more than four convex lenses and 14 cylindrical lenses of positive focal length are required to realize any matrix in $Sp(4,\mathbb{R})$. Besides, if the symplectic matrix $\mathbf{S} \in Sp(4,\mathbb{R})$ is of the form $\mathbf{S} = S_1 \oplus S_2$, where $S_1,\ S_2 \in Sp(2,\mathbb{R})$, then we prove that such $\mathbf{S}$ can be realized using not more than three convex lenses and seven cylindrical lenses of positive focal length. In addition, we have provided for the first time, to the best of our knowledge, explicit decompositions for the well known optical transformations such as differential free and inverse propagations, partial and inverse Fourier transformations, image reflectors, differential magnifiers, and differential fractional Fourier transformations.

Having realized any general FOOS, we now consider the problem of retrieving the phase possessing dislocations from two or more transverse plane intensity measurements. In this regard, we propose an iterative algorithm to retrieve phases with dislocations through three transverse plane intensity measurements and demonstrate the same numerically using Monte-Carlo simulations. The algorithm makes use of partial Fourier transformation, i.e., Fourier transformation in one transverse coordinate and identity transformation on the other, an example of asymmetric FOOS which is optically implemented using thin cylindrical lenses and free propagation transformations. Because partial Fourier transforma-

tion, unlike Fourier transformation, does not conserve longitudinal orbital angular momentum (OAM) of the wave field in general, the phase dislocation(s) possessed by the wave field is (are) created or destroyed naturally and its (their) orientation is (are) also distinguished. For numerical demonstration, we consider complex random linear superpositions of Laguerre-Gaussian (LG) beams with added white Gaussian noise. In all studied random superpositions, the algorithm converged well within 100 iterations. The proposed method is found to work well for wave fields with both integral and non-integral longitudinal OAM.

We then deal with the problem of phase retrieval from perturbed straight line fringes. For this problem, we present a single-shot phase retrieval algorithm from straight line fringes without phase unwrapping. It is worth mentioning here that most of the phase retrieving algorithms available in the literature require phase unwrapping – a process in which the extracted phase has to unwrapped, i.e., add and subtract 2π wherever discontinuity occurs, typically owing to the periodicity of $\tan^{-1}$ function. We first obtain phase gradients from the given interferogram using Hilbert-pair method, and then the desired phase is estimated from the gradients using the method of least squares for the Hudgin geometry. Here, the inverse of the matrix appearing in the method of least squares is computed analytically through the use of discrete cosine transform (DCT) as well as additional symmetries available in the Hudgin geometry. As a result, the computational time is reduced drastically and could as well be implemented in real-time applications. The algorithm is tested on both numerical interferograms with added white Gaussian noise, as well as on interferograms obtained from a Mach–Zehnder setup, where the respective imparted phases were random, and corresponded to atmospheric turbulence-like models.