Higher-Order High-Resolution Schemes for Hyperbolic Equations

A thesis submitted
in partial fulfillment for the degree of

Doctor of Philosophy

by

ARUN GOVIND NEELAN A

DEPARTMENT OF AEROSPACE ENGINEERING

INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY

THIRUVANANTHAPURAM - 695547, INDIA

JULY 2021

Abstract

An accurate resolution of shock structures is crucial to the design and load estimation of objects flying beyond sound speed. The aero-thermal loads acting on objects cruising at supersonic speed are very sensitive to the Mach number. Most spacecraft use a very low safety factor. Accurate resolution of shock structures is essential for an optimal and efficient space mission. High-resolution and higher-order schemes are needed to resolve shock structures without much dissipation and dispersion. Higher-order schemes are generally computationally expensive per iteration but are computationally economical for a given error. In addition to that higher-order schemes can able to resolve complex vortex structures relatively better than lower-order schemes.

Some of the novel schemes with high-resolution higher-order properties are presented in this work. These new schemes are computationally cost-effective than other schemes. A new family of Runge-Kutta (RK) method is presented. They are optimized for stability using an evolutionary algorithm and are suitable for shock-related problems. They outperformed the classical, and strong stability Runge-Kutta method in terms of stability and convergence. The procedures for obtaining higher-order schemes in uniform and non-uniform mesh are explored. Conservative discretization leads to lower-order convergence when a non-uniform grid is used. They can give a good convergence rate when a progressivly streched grid is used. This phenomenon is analytically proved using symbolic based code..

A second-order and a third-order limiter with excellent shock-resolution properties are presented. They are ideal for problems with blast waves. In terms of shock-resolving

properties, they outperformed other limiters. High-resolution schemes such as weighted essentially non-oscillatory schemes (WENO) and limiter schemes are explored, and novel schemes are presented. The classical WENO schemes are two-level schemes because they stay at either third-order or fifth-order. The current scheme is a three-level scheme that can deliver third-order, fourth-order and fifth-order accuracy. The current scheme has a tuning parameter that can be tuned based on the problem requirement. The current scheme outperformed other WENO schemes considered in this paper based on resolution and computational costs. A hybrid shock capture-fitting algorithm that can solve a shock with three grid points is presented. Some of the standard and non-standard schemes are explored in this work and have observed some significant improvements in terms of computational costs and shock resolution property.