A STOCHASTIC WAVELET FINITE ELEMENT METHOD USING B-SPLINE WAVELET ON THE INTERVAL FOR PROBLEMS IN STRUCTURAL MECHANICS

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by

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Abstract

A stochastic modelling approach takes into account the inherent uncertainties during the design of engineering systems. The additional statistical information that a stochastic model provides when compared with a deterministic model, leads to a comprehensive description and optimum design of the system; thereby improving its reliability. On the other hand, a stochastic approach also increases the model complexity and requires a higher computational effort to obtain the system response when compared with a deterministic approach. Numerical method such as B-spline wavelet on the interval (BSWI) based wavelet finite element method (WFEM) has shown to be efficient in dealing with the issues of low convergence as encountered in a conventional finite element method (FEM). At the same time, the underlying properties of wavelets could also be used to develop algorithms in the stochastic framework which could alleviate the issues related to accuracy and mapping of random field mesh that are encountered in FEM.

Based on the preceding notion, the current thesis at first proposes a background cell based Gauss quadrature numerical integration approach for BSWI WFEM. In the proposed approach, background cells are placed over each BSWI element and Gauss quadrature rule is defined for each of these cells. During the analysis, background cells of various lengths are used for evaluating the integrals for various combination of order and resolution of BSWI scaling functions. The dimensions of the background cells are varied and its effect on the condition number and sparseness of the element stiffness matrix is studied for one dimensional (1D) (bar, beams) and two dimensional (2D) plane stress problems. Further, a detailed analysis to understand the effect of number of Gauss points within each background cell on the accuracy of the results is done.

The development of stochastic BSWI WFEM algorithms for linear static problems in 1D (bar), beams (based on Euler-Bernoulli and Timoshenko beam theory) and 2D plane elasto-statics (plane stress) is shown in the thesis wherein, the spatial variation of modulus of elasticity is modelled as a homogeneous random field. BSWI scaling functions are used for the discretization of the random field. The response statistics are obtained using the perturbation approach. Numerical examples are solved based on the proposed background cell integration scheme. The results from perturbation approach are compared with that obtained from Monte Carlo simulation (MCS). A parametric study is also done

to understand the effect of coefficient of variation values and correlation length parameters on the response statistics. For 1D problem in particular, results from proposed stochastic WFEM method are compared with those found using stochastic FEM wherein random field discretization is done using Lagrange shape functions. Furthermore, normalized computational times for the execution of perturbation approach and MCS based on WFEM are evaluated and compared with those obtained for FEM.

In addition to the aforementioned formulations, the thesis shows the construction of beam elements by incorporating von Kármán nonlinear strains using BSWI WFEM. The mathematical model is developed in both the deterministic as well as the stochastic framework. An algorithm for evaluating the derivatives of response quantities from nonlinear equilibrium equations is derived. The results are analyzed accordingly for different boundary conditions. Results show that BSWI WFEM can be used as an alternate numerical tool for developing an efficient and rigorous stochastic based algorithms.