Enhanced Sparsity Order Estimation Techniques for Dynamic Compressed Sensing

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Abstract

We present a composite Compressed Sensing (CS) system for the efficient acquisition and recovery of sparse and compressible time-varying signals. Sparsity Order Estimation (SOE) is critical in determining the efficiency of CS acquisition and recovery because the sparsity order parameter of an underlying signal plays an important role in determining the number of CS samples or measurements to be obtained from the underlying signal during acquisition and the number of iterations required to recover the support of the underlying signal during recovery. As a result, the proposed composite CS system is built by vertically stacking a sparse Binary Sensing Matrix (BSM) and a dense Gaussian Sensing Matrix (GSM), with the sparse BSM assisting the SOE during acquisition and recovery and the GSM assisting reconstruction during recovery. The proposed sparse BSM is deterministic and adapts to the sparsity order variations for an efficient SOE. The GSM is dense and random, and satisfies the Restricted Isometry Property (RIP) with an overwhelming probability for guaranteed recovery. Because of the BSM's weaker RIP, we limit the number of BSM-based measurements.

We propose the SOE based on two different Maximum Likelihood (ML) principles: (i) BSM-based SOE (BSOE), which takes advantage of the sparse structure of the BSM and the statistics of BSM-based measurements, and (ii) GSM-based SOE (GSOE), which takes advantage of the statistics of GSM-based measurements. The proposed BSOE method does not require any prior knowledge of the underlying signal, but it estimates the statistics of the underlying signal, and thus the sparsity order, with a limited number of measurements. To perform the SOE, the GSOE method requires statistical estimates obtained from the BSOE method. We demonstrate that both ML estimation methods produce unbiased estimates, and that the GSOE method meets the Cramer-Rao Lower Bound.

As the sparsity order varies over time owing to the continuous birth of newer supporting components and the death of existing supporting components, we characterize the sparsity order variation as a stochastic Markov birth-death process. A statistical measure, namely, survival time, is introduced here to statistically quantify the degree and duration of invariance of the sparsity order. We then refine the ML estimates of sparsity order using either of the two independent approaches, namely, Viterbi algorithm-based ML sequence estimation and Kalman filtering of ML estimation by exploiting the underlying discrete Markov process and the survival time, which characterizes the sparsity order variation.

We develop a BSM Aided Orthogonal Matching Pursuit (BAOMP) method for the faster recovery of sparse and compressible signals. Although the sparse BSM is limited owing to its weaker isometry bounds, each BSM-based measurement provides initial estimates of the probable support candidates based on the location of those in the corresponding BSM row. Because of these initial estimates, the number of iterations required for the recovery algorithm is subsequently reduced, and the speed of recovery is improved by at least 25% compared to existing recovery methods.

The proposed composite CS, ML sparsity order estimators, and BAOMP-based recovery algorithms are practically implementable and can be used in real-world applications such as (i) vibration signal acquisition and recovery, (ii) channel estimation, and (iii) electro cardiogram signal recovery. The proposed method's performance is then compared to existing methods using metrics such as SOE error, Normalized Recovery Error, and run-time complexities. The results on real-world and synthetic data show that the proposed methods work with better performances, even with a low signal-to-noise ratio.