Virtual Element Method for Time Dependent Problems on Polygonal Meshes

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by

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Abstract

This work is concerned with the numerical approximation of time dependent partial differential equations(PDE) in the context of virtual element method(VEM). Basic aspects of VEM are revealed in Chapter-1 dissecting elliptic PDE. Fundamental theoretical framework as well as computational aspect of VEM are developed based on two reliable projection operators-L^2 projection operator $\Pi^0_{k,K}$ and energy projection operator $\Pi^{\nabla}_{k,K}$(defined locally). Basis functions are constructed virtually, can be thought of a solution of some PDEs which determine the dimension of virtual element spaces. The method is designed in such a way that it does not require explicit information about basis functions. Informations provided by degrees of freedom (DoF) are enough to evaluate basis functions that exempt us from hectic polynomial integration. Moreover, a polynomial spaces sitting inside VEM space ensure optimal order of convergence. The second but not secondary advantage of VEM is that discrete formulation satisfies Galerkin approximation like FEM. Hence, many fundamental properties of FEM can be incorporated in VEM. In Chapter-1, we briefly study basic properties of VEM and computation of projection operators $\Pi^0_{k,K}$ and $\Pi^{\nabla}_{k,K}$ (defined in same chapter). Moreover, using analogous idea form [1], we modify the virtual element space without changing DoF that ensure computation of L^2 projection operator $\Pi^0_{k,K}$.

In Chapter-2 and Chapter-3, we propose a numerical method to approximate semi-linear parabolic and hyperbolic equations respectively. The method is designed by exploiting L^2 projection operator $\Pi^0_{k,K}$ in order to evaluate non-linear load term. We consider modified virtual element space for ensuring optimal order of convergence in H^1 and L^2 norm for semi-discrete and fully-discrete schemes. Furthermore, fully-discrete scheme reduces to non-linear system of equations which can be solved by employing Newton method. Since, Newton method is computationally expensive, we present linearised scheme that ensures optimal order of convergence. For time discretization, we employ backward Euler scheme and Newmark scheme for parabolic and hyperbolic equations respectively. We have conducted numerical experiments on polygonal meshes to illustrate the performance of the proposed scheme and validate the theoretical findings.

In Chapter-4, we have studied the convection dominated diffusion reaction equation using SUPG stabilizers for both FEM and VEM. We initiate our discussion by exploring new finite element for convection dominated diffusion reaction equations. We con-

sider two different bilinear forms and examine convergence behaviour in mesh-dependent norm. Numerical experiments were conducted to emphasize theoretical results. Later, we extend this discussion for time dependent convection dominated diffusion reaction equation in the context of VEM. In the present scheme, VEM is used for space discretization whereas Crank-Nicolson scheme is employed for time discretization. Since the model problem is convection dominated, we add additional SUPG type stabilizer in order to obtain stable numerical solutions. Both semi and fully discretize schemes are analyzed and convergence analysis is carried out in mesh dependent norm and in L^2 norm. A set of numerical examples is presented in order to judge the computational efficiency of the proposed scheme and also to validate our theoretical findings.

In Chapter-5, we address time dependent Stokes equation using VEM. Velocity is approximated using $H1$ conforming discrete inf-sup stable virtual element space and pressure is approximated by discontinuous piecewise polynomial space. We approximate non-stationary part and right hand side load term exploiting vector valued L^2 projection operator. Following analogous idea as [2], we modify the VEM space such that L^2 projection operator $\Pi^0_{k,K}$ is computable. Moreover, we introduce discrete Stokes projection to pursue convergence analysis for semi-discrete case.

In light of the above works, we have drawn some conclusions in Chapter-6 and pointed out some future works in same chapter.