

**DISCONTINUOUS FINITE VOLUME  
METHODS FOR OPTIMAL CONTROL  
PROBLEMS**

*A thesis submitted  
in partial fulfillment for the degree of*

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*by*

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## ABSTRACT

The main objective of this thesis is to develop and analyze discontinuous finite volume methods for the approximation of distributed optimal control problems governed by certain partial differential equations subject to pointwise control constraints. In view of applications, we consider optimal control problems governed by semilinear elliptic, parabolic and hyperbolic problems, and Brinkman equations (that describe flow of an incompressible viscous fluid through a porous medium). For the discretization of state and costate variables, we utilize piecewise linear discontinuous finite volume schemes, whereas three different strategies are used for control approximation: variational discretization approach—in which control set is not discretized explicitly but discretized by a projection of the discrete costate variables, as well as piecewise constant and piecewise linear discretizations. As the resulting discrete optimal systems are non-symmetric, we employ the so-called *optimize-then-discretize* approach to approximate the control problem. *A priori* error estimates in suitable natural norms are derived for control, state and costate variables. Further, numerical experiments are presented to illustrate the performance of the proposed schemes and to confirm the predicted accuracy of the theoretical convergence rates. Finally, based on theoretical and computational observations, this thesis addresses concluding remarks and future work regarding possible extensions of present work to more application based and real life problems.