LOWER DIMENSIONAL APPROXIMATION OF THIN ELASTIC AND PIEZOELECTRIC SHELLS

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by

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ABSTRACT

In this thesis, we first derive the lower dimensional model of linear elastic shallow shells using gamma convergence. We then justify the scalings used to derive the two dimensional model and finally we derive the two dimensional approximation of thin piezoelectric shallow shells with variable thickness.

In chapter 2, we consider the case of linear elastic shallow shells. Here we consider a boundary value problem in three dimensional elasticity posed over a shell of thickness 2ϵ having a specific geometry and clamped on a portion of its lateral surface. We then transfer the problem to a domain independent of the thickness parameter by suitable scalings on the unknowns and data and we show that the energy functionals $J(\epsilon)$ of the three-dimensional problem gamma converges to the energy functional associated with the two-dimensional problem and hence the sequence of functions which minimizes the energy associated with the three dimensional problem converge weakly to the function which minimizes the energy associated with a two dimensional model.

In chapter 3, we justify the scalings on the unknowns and data used to derive the two dimensional model of linearly elastic shallow shells.

The method of asymptotic analysis for deriving the two-dimensional models of plates and shells rely in a crucial way on appropriate scalings of the components of the displacement and appropriate assumptions on the data (Lamé constants and applied forces). The question is “are these scalings unique”?

This leads to the question of justifying the scalings used to derive these lower dimensional models.

We apply the formal asymptotic method to the variational formulation of the three-dimensional boundary value problems of linear shallow shells. Without making any *a priori* assumption of a mechanical or geometrical nature, we provide a complete justification of the scalings and assumptions.

In chapter 4, we consider thin piezoelectric shells with *variable* thickness. We first pose the problem in variational form and transfer the problem, by making suitable scalings on the unknowns and data, to a domain which is independent of the thickness parameter. We then show that the scaled solutions converge to a solution of a two dimensional model.