INVESTIGATING LIMIT CYCLE PERFORMANCE AND ASYMPTOTIC BODE BEHAVIOR OF FRACTIONAL-ORDER CONTROLLERS

A thesis submitted
in partial fulfillment for the degree of

Doctor of Philosophy

by

AMEYA ANIL KESARKAR

Department of Avionics
INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY
Thiruvananthapuram - 695547

May 2015

ABSTRACT

Fractional Calculus (FC) is a branch of mathematics which generalizes classical integer-order calculus to handle integrals and derivatives of *arbitrary* orders. Recently, the FC has received attentions in various science and engineering fields including control theory. In control theory, one deals with the design and analysis of Fractional-Order Controllers (FOCs), whose dynamics are governed by fractional-order differential equations. In this thesis, our main objective is to investigate the limit cycle performance and asymptotic Bode characteristics of such FOCs. We also derive unified tuning expressions for three-parameter FOCs which meet Wang-et-al specifications.

The thesis begins by considering the unification of tuning expressions for three-parameter FOCs such as PI^{α}, $[PI]^{\alpha}$, PD^{β}, and $[PD]^{\beta}$ to meet desired gain crossover frequency, phase margin and isodamping property (Wang-et-al specifications) with the help of a proposed universal plant structure.

Then, we focus on the plants containing separable nonlinearity and observe the limit cycle suppression capabilities of FOCs when they are tuned for the Transfer Function (TF) of such plants to meet the Wang-et-al specifications. A typical motion-servo plant containing separable backlash nonlinearity is considered for this purpose and three-parameter FOCs such as PI^{α}, $[PI]^{\alpha}$ and integer PID are tuned by using our earlier derived unified expressions. When the limit cycle performances of such controllers are examined in the presence of plant nonlinearity, it is found that the FOCs remarkably suppress amplitude of limit cycles than the integer PID which subsequently results into lesser amplitude sustained oscillations in the steady state of closed loop response. This is further justified using Describing Function (DF) analysis method. It is noticed that the reason for such distinct performance lies in the location of intersection point corresponding to Nyquist condition for limit cycles. The confirmation of such fractional superiority is further made for the Precision Modular Servo (PMS) laboratory set-up under the similar tuning conditions.

Motivated by the above simulation and experimental studies, a more detailed in-

vestigation is pursued towards suppressing the sustained oscillation amplitudes for two kinds of plants, one containing backlash and the other with relay nonlinearity. For each plant, the controller design is formulated as a constrained optimization problem to obtain the desirable limit cycle performance. Additionally, the controller is forced to meet certain loop performance specifications. The DF of the nonlinearity is efficiently utilized during the construction of these frameworks. Under such novel formulation, the superiority of FOCs over their integer-order counterparts is investigated in detail.

In the thesis, we further contribute towards characterizing the asymptotic Bode behavior of FOCs such as PI^α, $[PI]^\alpha$, PD^β, $[PD]^\beta$, and $PI^\alpha D^\beta$. The work introduces a few basic fractional-order terms for this purpose and develops their asymptotic magnitude and phase Bode plots. Later, such plots are utilized in constructing the asymptotic magnitude and phase Bode plots for the said FOCs. We also develop such plots for the fractional commensurate order TFs in general. Identification of fractional-order TF from the given asymptotic magnitude Bode plot is illustrated in detail. Additionally, the application of asymptotic magnitude and phase Bode plots for analyzing a given fractional control loop is also explained using a numerical example.